

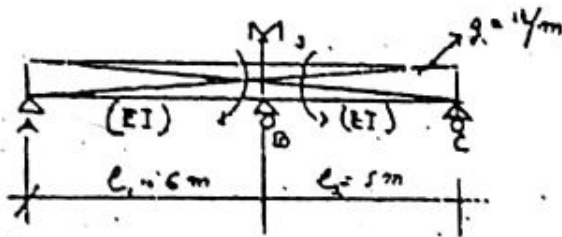
A. remakalan dalil 3 Momen Clapayron pada konstruksi batang da - tar.

Prinsip : Sudut belahan yang terjadi karena adanya muatan = sudut belahan yang terjadi karena momen reaksi.

1.

Diketahui : Konstruksi ter-gambar.

Ditanyakan: a) bid M
b) bid D



Jawab:

$$\text{a) Karena muatan: } \alpha_{b_1} + \alpha_{b_2} = \frac{q l_1^3}{24EI} + \frac{q l_2^3}{24EI} = \frac{216}{24EI} + \frac{125}{24EI} = \frac{341}{24EI}$$

$$\text{Karena momen } M_B: \beta_{b_1} + \beta_{b_2} = \frac{M_B l_1}{3EI} + \frac{M_B l_2}{3EI} = \frac{6 M_B + 5 M_B}{3 EI} = \frac{11 M_B}{3EI}$$

$$\frac{341}{24EI} = \frac{11 M_B}{3EI} \rightarrow \frac{341}{8} = 11 M_B \rightarrow M_B = 3,88 \text{ tm } (+)$$

∴ Dalam gambar tandanya (-)

Muatan dianggap terletak diatas batang sendi - rol :

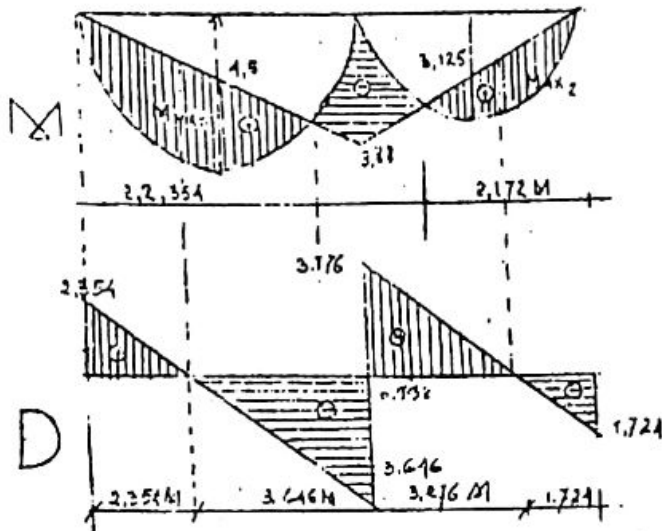
$$M'_{\max_1} = 1/8 q l_1^2 = \frac{36}{8} = 4 \frac{1}{2} \text{ tm} = 4,5 \text{ tm}$$

$$M'_{\max_2} = 1/8 q l_2^2 = \frac{25}{8} = 3 \frac{1}{8} \text{ tm} = 3,125 \text{ tm}$$

$$b) R_A = 3t - \frac{M_B}{6} = 3t - \frac{3,88}{6} t = 3t - 0,646 t = 2,354 t.$$

$$R_D = 3t + 2,5t + \frac{M_B}{5} = 5,5t + \frac{3,88}{5} t + \frac{3,88}{5} t = 6,922 t.$$

$$R_C = 2,5t - \frac{M_B}{5} = 2,5t - \frac{3,88}{5} t = 1,724 t.$$



$$M_{max_1} = 1/8 q (1')^2 =$$

$$1/8 q (2.2,354)^2 =$$

$$M_{max_2} = 1/8 q (1')^2 =$$

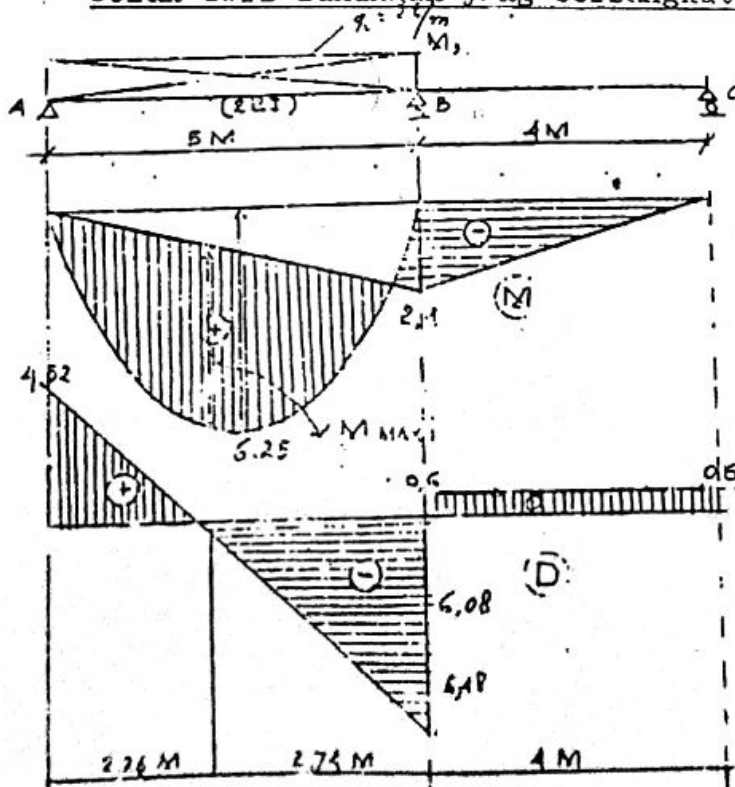
$$1/8 q (2.1,724)^2.$$

Pembuatan bidang D :

Besarnya reaksi suatu dukungan = besarnya reaksi jika muatannya dianggap terletak diatas batang sendi - rol biasa (Statis Tertentu) + besarnya reaksi karena pengaruh momen M.

Besarnya reaksi karena pengaruh momen ini = $\frac{M}{I}$.

Jika momen M pada dukungan itu sendiri maka $\frac{M}{I}$ bertanda positif (+) dan jika momen M pada dukungan yang ada disebelah dari dukungan yang bersangkutan maka $\frac{M}{I}$ itu bertanda negatif (-).



Diketahui: Konstruksi tergambar.

Ditanyakan: a) bid M
b) bid D

Jawab:

a) Krn muatan: $\alpha_{b_1} +$

$$\alpha_{b_2} = \frac{q l^3}{24(2EI)} + 0 = \frac{2.125}{48EI} = \frac{125}{24EI}.$$

Krn momen: $\beta_{b_1} + \beta_{b_2} =$

$$\frac{M_B \cdot 1_1}{3(2EI)} + \frac{M_B \cdot 1_2}{3EI} = \frac{5M_B}{6EI} + \frac{4M_B}{3EI} = \frac{13M_B}{6EI}$$

$$\therefore \frac{125}{24EI} = \frac{13M_B}{6EI} \rightarrow M_B = \frac{6.125}{13.24} = 2.4 \text{ tm.}$$

$$M'_{max} = 1/8 q l^2 = 1/8.2.15 = 6,25 \text{ tm.}$$

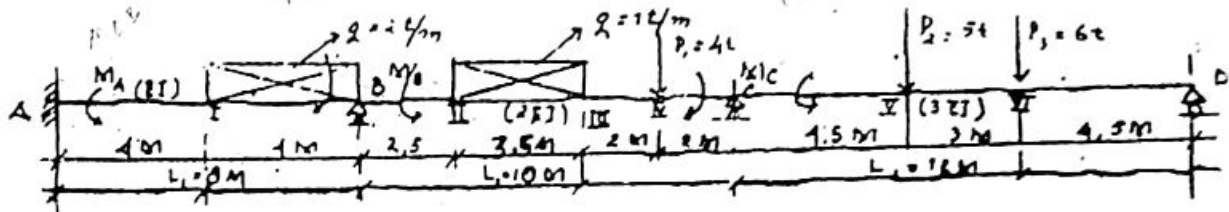
$$b) R_A = 5t - \frac{M_B}{5} = 5t - \frac{2,4}{5} = 4,52 \text{ t.}$$

$$R_B = 5t + \frac{M_B}{5} + \frac{M_B}{4} = -5 + \frac{2,4}{5} + \frac{2,4}{4} = 6,03 \text{ t.}$$

$$R_C = -\frac{M_B}{4} = -\frac{2,4}{4} = -0,6$$

$$M_{\max} = 1/8 q (1')^2 = 1/8 \cdot 2 \cdot (2,2,26)^2.$$

3.



Diketahui : Konstruksi tergambar.

Ditanyakan: a) bid. M. b) bid. D.

Jawab:

Karena muatan: $\alpha_a = \frac{7}{384} \cdot \frac{q_1 l_1^3}{EI} = \frac{7}{384} \cdot \frac{2,8^3}{EI} = \frac{18,67}{EI}$

$$\alpha_{b_1} + \alpha_{b_2} = \frac{9}{384} \cdot \frac{q_1 l_1^3}{EI} + \frac{q_2}{6(2EI)l_2}$$

$$\left| \frac{1}{2} l_2^2 x^2 - \frac{1}{4} x^4 \right|_{4}^{7,5} + \frac{P_1 \cdot 2(l_2^2 - 2^2)}{6(2EI)l_2}$$

$$= \frac{9}{384} \cdot \frac{2,8^3}{EI} + \frac{1}{120EI} \left| 50x^2 - \frac{1}{4} x^4 \right|_{4}^{7,5} + \frac{4 \cdot 2(100-4)}{120EI}$$

$$= \frac{24}{EI} + \frac{1}{120EI} \left\{ 50 \cdot 7,5^2 - \frac{1}{4} \cdot 7,5^4 \right\}$$

$$- \left(50 \cdot 4^2 - \frac{1}{4} \cdot 4^4 \right) \left\} + \frac{6,4}{EI}\right.$$

$$= \frac{24}{EI} + \frac{10,74}{EI} + \frac{6,4}{EI} = \frac{41,14}{EI}$$

$$\alpha_{c_1} + \alpha_{c_2} = \frac{q_2}{6(2EI)l_2} \left| \frac{1}{2} l_2^2 x^2 - \frac{1}{4} x^4 \right|_{2,5}^6 + \frac{P_1 \cdot 8(l_2^2 - 8^2)}{6(2EI)l_2}$$

$$+ \frac{P_2 \cdot 7,5(l_2^2 - 7,5^2) + P_3 \cdot 4,5(l_2^2 - 4,5^2)}{6(3EI)l_3}$$

$$= \frac{1}{120EI} \left| 50x^2 - \frac{1}{4} x^4 \right|_{2,5}^6 + \frac{4 \cdot 8(100 - 64)}{120EI}$$

$$+ \frac{5 \cdot 7,5(144 - 56,25) + 6 \cdot 4,5(144 - 20,25)}{216EI}$$

$$= \frac{1}{120EI} \left\{ (50 \cdot 6^2 - \frac{1}{4} \cdot 6^4) - (50 \cdot 2,5^2 - \frac{1}{4} \cdot 2,5^4) \right\} + \frac{9,61}{EI} + \frac{30,7}{EI}$$

$$= \frac{9,78}{EI} + \frac{9,61}{EI} + \frac{30,7}{EI} = \frac{50,09}{EI}$$

$$\text{Krn momen: } \beta_a = \frac{M_A \cdot 1_1}{3EI} + \frac{M_B \cdot 1_1}{6EI} = \frac{8 \cdot M_A}{3EI} + \frac{8 \cdot M_B}{6EI} = \frac{2,67M_A}{EI} + \frac{1,33M_B}{EI}$$

$$\beta_{b_1} + \beta_{b_2} = \frac{M_A \cdot 1_1}{6EI} + \frac{M_B \cdot 1_1}{3EI} + \frac{M_B \cdot 1_2}{3(2EI)} + \frac{M_C \cdot 1_2}{6(2EI)} = \frac{8M_A}{6EI} + \frac{8M_B}{3EI} + \frac{10M_B}{6EI} + \frac{10M_C}{12EI}$$

$$= \frac{1,33M_A}{EI} + \frac{4,33M_B}{EI} + \frac{0,833M_C}{EI}$$

$$\beta_{c_1} + \beta_{c_2} = \frac{M_B \cdot 1_2}{6(2EI)} + \frac{M_C \cdot 1_2}{3(2EI)} + \frac{M_C \cdot 1_3}{3(3EI)} = \frac{10M_B}{12EI} + \frac{10M_C}{6EI} + \frac{12M_C}{9EI}$$

$$= \frac{0,833M_B}{EI} + \frac{3M_C}{EI}$$

Pers. belahan yang terjadi :

$$\alpha_a = \beta_a \rightarrow \frac{18,67}{EI} = \frac{2,67M_A}{EI} + \frac{1,33M_B}{EI} \rightarrow \therefore 2,67M_A + 1,33M_B = 18,67 \quad (I)$$

$$\alpha_{b_1} + \alpha_{b_2} = \beta_{b_1} + \beta_{b_2} \rightarrow \frac{41,14}{EI} = \frac{1,33M_A}{EI} + \frac{4,33M_B}{EI} + \frac{0,833M_C}{EI}$$

$$\rightarrow \therefore 1,33M_A + 4,33M_B + 0,833M_C = 41,14 \quad (II)$$

$$\alpha_{c_1} + \alpha_{c_2} = \beta_{c_1} + \beta_{c_2} \rightarrow \frac{50,09}{EI} = \frac{0,833M_B}{EI} + \frac{3M_C}{EI}$$

$$\rightarrow \therefore 0,833M_B + 3M_C = 50,09 \quad (III)$$

Jadi terdapat 3 persamaan dengan 3 bil. yang tak diketahui M_A , M_B dan M_C . Maka M_A , M_B dan M_C itu dapat dicari :

$$4,79 M_A + 15,6 M_B + 3 M_C = 148,2 \quad (II)$$

$$0,833M_B + 3M_C = 50,09 \quad (III)$$

$$4,79 M_A + 14,767 M_B = 98,11$$

$$4,79 M_A + 14,767 M_B = 98,11$$

$$29,64 M_A + 14,767 M_B = 207,2 \quad (I)$$

$$- 24,85 M_A = - 109,09 \rightarrow M_A = \frac{109,09}{24,85} = 4,39 \text{ tm.}$$

$$(I) \quad 2,67M_A + 1,33M_B = 18,67$$

$$\therefore 2,67 \cdot 4,39 + 1,33M_B = 18,67 \rightarrow M_B = \frac{6,95}{1,33} = 5,225 \text{ tm.}$$

$$(III), 0,833M_B + 3M_C = 50,09$$

$$\therefore 0,833 \cdot 5,225 + 3M_C = 50,09 \rightarrow M_C = \frac{46,74}{3} = 15,58 \text{ tm.}$$

Muatan dianggap terletak diatas batang sendi - rol:

1) Perhatikan konstruksi AB : $\bar{R}_A = \frac{8 \cdot 2}{8} = 2t$. $\bar{R}_{B_1} = \frac{8 \cdot 6}{8} = 6t$.

$$\bar{M}_I = \bar{R}_A \cdot 4 = 2 \cdot 4 = 8 \text{ tm.}$$

$$M_x = R_{B_1} \cdot x - \frac{1}{2} q x^2 = 6x - x^2.$$

$$M'_{\text{max}} \text{ dicapai bila } \frac{dM_x}{dx} = 0 \rightarrow 6 - 2x = 0 \rightarrow \therefore x = 3m.$$

$$\therefore M'_{\text{max}} = 6 \cdot 3 - 3^2 = 9 \text{tm. berjarak } 3m \text{ dikiri B.}$$

2) Perhatikan Konstruksi BC : $R_{B_2} = \frac{3,5 \cdot 5,75 + 4 \cdot 2}{10} = 2,875t$.

$$R_{C_1} = \frac{3,5 \cdot 4,25 + 4 \cdot 8}{10} = 4,625t.$$

$$\bar{M}_{II} = \bar{R}_{B_2} \cdot 2,5 = 2,875 \cdot 2,5 = 7,19 \text{ tm.}$$

$$\bar{M}_{III} = \bar{R}_{C_1} \cdot 4 - 4 \cdot 2 = 4,625 \cdot 4 - 8 = 10,5 \text{ tm.}$$

$$\bar{M}_{IV} = \bar{R}_{C_1} \cdot 2 = 4,625 \cdot 2 = 9,25 \text{ tm.}$$

$$M_x = R_{B_2} (2,5 + x) - \frac{1}{2} q x^2 = 2,875(2,5 + x) - \frac{1}{2} q x^2 = 7,19 + 2,875x - \frac{1}{2} x^2.$$

$$M'_{\text{max}} \text{ dicapai bila } \frac{dM_x}{dx} = 0 \rightarrow 2,875 - x = 0 \rightarrow \therefore x = 2,875.$$

$$\therefore M'_{\text{max}} = 7,19 + 1,875 \cdot 2,875 - \frac{1}{2} \cdot (2,875)^2 = 11,32 \text{ tm. berjarak } (2,5 + 2,875)m \text{ dikanan B.}$$

3) Perhatikan konstruksi CD : $R_{C_2} = \frac{5 \cdot 7,5 + 6 \cdot 4,5}{12} = 5,375t$.

$$R_D = \frac{5 \cdot 4,5 + 6 \cdot 7,5}{12} = 5,625t.$$

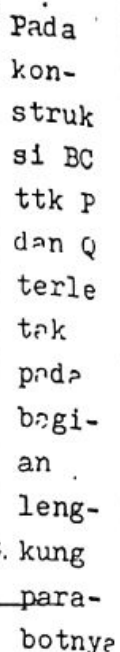
$$\bar{M}_V = \bar{R}_{C_2} \cdot 4,5 = 5,375 \cdot 4,5 = 24,2 \text{tm. } \bar{M}_{VI} = \bar{R}_D \cdot 4,5 = 5,625 \cdot 4,5 = 25,34 \text{ tm.}$$

b) $R_A = \bar{R}_A + \frac{M_A}{1} - \frac{M_B}{1} = 2t + \frac{4,39}{8} - \frac{5,225}{8} = 1,896t$.

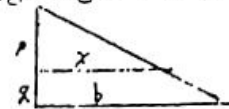
$$R_B = R_{B_1} + R_{B_2} - \frac{M_A}{1} + \frac{M_B}{1} + \frac{M_C}{1} = 6t + 2,875t - \frac{4,39}{8} + \frac{5,225}{8} + \frac{5,225}{10} = \frac{15,58}{10}t = 7,9445t.$$

$$R_C = \bar{R}_{C_1} + \bar{R}_{C_2} - \frac{M_B}{1} + \frac{M_C}{1} + \frac{M_D}{1} = 4,625t + 5,375t - \frac{5,225}{10} + \frac{15,58}{10}t + \frac{15,58}{12}t = 12,3355t.$$

$$R_D = \bar{R}_D - \frac{M_C}{1} = 5,625t - \frac{15,58}{12}t = 4,325t.$$



Untuk mencari momen max itu diperlukan persamaan pada trapesium (lihat samping) : $X = \frac{pb + qa}{p + q}$ atau pada segitiga : $X = \frac{p \cdot b}{p + q}$



$$x = \frac{p \cdot b}{p + \rho \cdot 4t}$$



$$R_A = 1,896t. \quad R_B = 7,944t. \quad R_C = 12,335t.$$

Berat muatan diatas konstruksi AB ialah : $2.4t = 8t$.

$$\therefore 1,896t + a = 8t. \rightarrow a = 8t - 1,896t = 6,104t.$$

$$\therefore d : e = c : a = 1,896 : 6,104 = 8.$$

$$\therefore d = \frac{1,896}{8} \cdot 4m = 0,948m. \quad e = \frac{6,104}{8} \cdot 4m = 3,052m.$$

$\therefore M_{max_1}$ terletak 4,948m dikanan A.

$$p = R_A \cdot 4,948 - \frac{1}{2} q x^2 = 2.4 \cdot 948 - \frac{1}{2} \cdot 2 \cdot (0,948)^2 = 8,997 \text{ tm}.$$

$$x = \frac{3,052 \cdot 4,39 + 4,948 \cdot 5,225}{8} = 4,9 \text{ tm}.$$

$$\therefore y = p - x = 8,997tm - 4,9tm = 4.097 \text{ tm}.$$

$$\therefore M_{max_1} = 4,097 \text{ tm}.$$

Berat muatan diatas konstruksi BC : $3,5t + 4t = 7,5t$.

$$b = 7,944t - a = 7,944t - 6,104t = 1,84t.$$

$$g = 7,5t - b = 7,5t - 1,84t = 5,66 \text{ t}.$$

$$f = g - 4t = 5,66t - 4t = 1,66t.$$

$$m : n = b : f = 1,84 : 1,66 = 3,5.$$

$$\therefore m = \frac{1,84}{3,5} \cdot 3,5m = 1,84m. \quad n = \frac{1,66}{3,5} \cdot 3,5m = 1,66m.$$

$\therefore M_{max_2}$ terletak 4,34 m dikanan B.

$$v = R_B \cdot 4,34 - \frac{1}{2} q x^2 = 2,875 \cdot 4,34 - \frac{1}{2} \cdot 1 \cdot (1,84)^2 = 10,77 \text{ tm}.$$

$$w = \frac{4,34 \cdot 15,58 + 5,66 \cdot 5,225}{10} = 9,71 \text{ tm}.$$

$$\therefore u = v - w = 10,77 \text{ tm} - 9,71 \text{ tm} = 1.06 \text{ tm}.$$

$$\therefore M_{max_2} = 1,06 \text{ tm}.$$

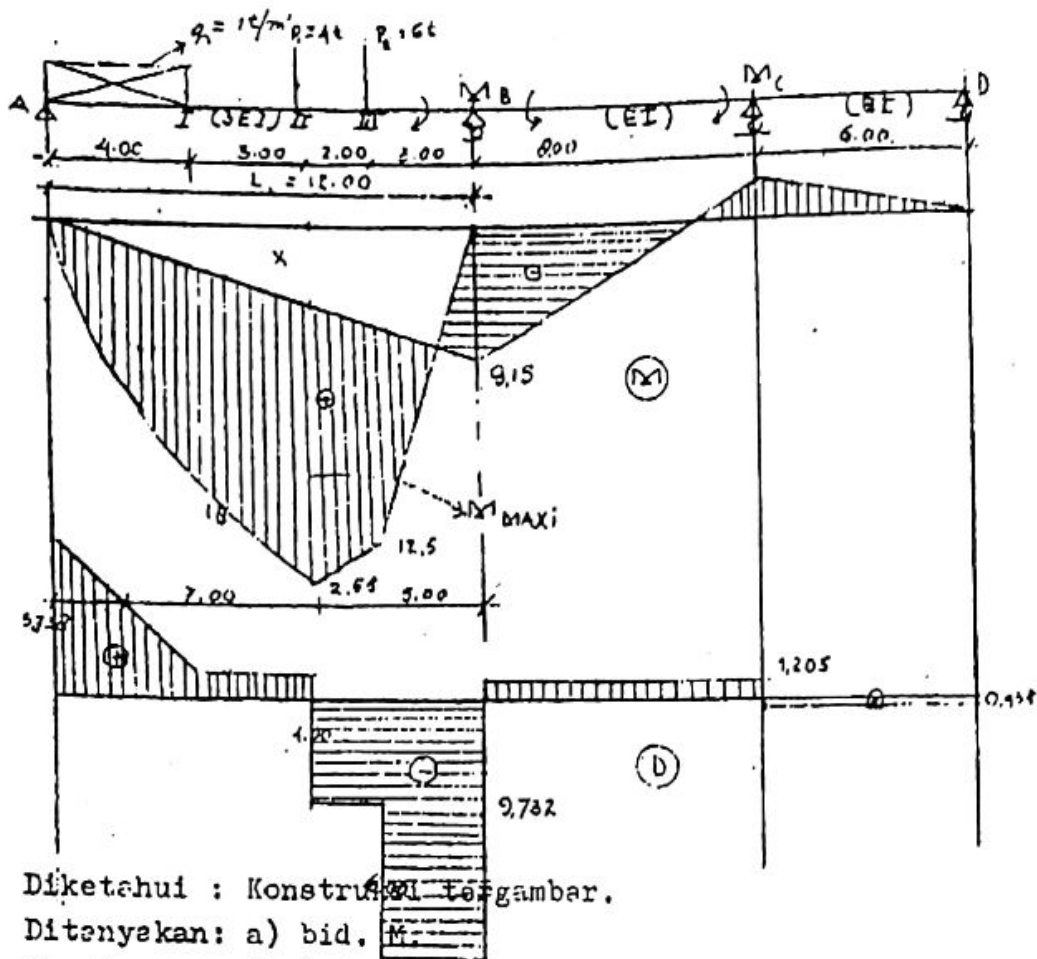
Dapatlah kita lihat diatas bahwa untuk mencari bes rnya M_{max} itu diperlukan gambar bid D dulu karena M_{max} itu terletak pada keadaan dimana $D = 0 \rightarrow$

$\therefore$ Jika letak $D = 0$ diketahui maka letak M_{max} diketahui.

Dalam penggambaran bid M dan bid D haruslah dengan Skala yang tepat/baik terhadap ukuran yang sebenarnya, karena kalau tidak akan membuat kesalahan dalam penggambarannya, ialah tentang letak titik potong dari superposisi bid M (misalnya).

Sebagai contoh diatas, ialah bahwa pada konstruksi BC bid M negatif memotong bid M positif pada bagian lengkung parabola (di-F dan Q).

Tapi kalau skalanya tak baik maka bid M negatif itu kemungkinan akan memotong bid M positif pada bagian yang lurus, yang berarti bahwa penggambaran ini salah.



Diketahui : Konstruksi gambar.

Ditanyakan: a) bid. M.

Jawab : a). bid. D.

a) Karena muatan:

$$\alpha_{b_1} + \alpha_{b_2} = \frac{q}{6(3EI)} \left[\frac{1}{2} 12x^2 - \frac{1}{4}x^4 \right]_0^4 + \frac{P_1 \cdot 7(1^2 - 7^2) + P_2 \cdot 9(1^2 - 9^2)}{6(3EI)} \frac{1}{2}$$

$$= \frac{1}{216EI} \left[72x^2 - \frac{1}{4}x^4 \right]_0^4 + \frac{4 \cdot 7(144 - 49) + 6 \cdot 9(144 - 81)}{216 \cdot EI}$$

$$= \frac{1}{216EI} (72 \cdot 4^2 - \frac{1}{4} \cdot 4^4) + \frac{28 \cdot 95 + 54 \cdot 63}{216 \cdot EI}$$

$$= \frac{5.04}{EI} + \frac{28.06}{EI} = \frac{33.1}{EI}$$

Karena momen:

$$\beta_{b_1} + \beta_{b_2} = \frac{M_B \cdot 11}{3(3EI)} + \frac{M_B \cdot 12}{3EI} + \frac{M_C \cdot 12}{6EI}$$

$$= \frac{12M_B}{9EI} + \frac{8M_B}{3EI} + \frac{8M_C}{6EI}$$

$$= \frac{4M_B}{EI} + \frac{1.33M_B}{EI}$$

$$\beta_{c_1} + \beta_{c_2} = \frac{M_B \cdot 12}{6EI} + \frac{M_C \cdot 12}{3EI} + \frac{M_C \cdot 13}{3EI}$$

$$= \frac{8M_B}{6EI} + \frac{8M_C}{3EI} + \frac{6M_C}{3EI}$$

$$= \frac{1.33M_B}{EI} + \frac{4.67M_C}{EI}$$

Persamaan belahan: $\alpha_{b_1} + \alpha_{b_2} = \beta_{b_1} + \beta_{b_2} : \alpha_{c_1} + \alpha_{c_2}$
 $= \beta_{c_1} + \beta_{c_2}$

$$\frac{33,1}{EI} = \frac{4M_B}{EI} + \frac{1,33M_C}{EI} \rightarrow 4M_B + 1,33M_C = 33,1 \dots\dots\dots (I).$$

$$0 = \frac{1,33M_B}{EI} + \frac{4,67M_C}{EI} \rightarrow 1,33M_B + 4,67M_C = 0 \dots\dots\dots (II).$$

$$14,05 M_B + 4,67 M_C = 116,2 \dots\dots\dots (I)$$

$$1,33 M_B + 4,67 M_C = 0 \dots\dots\dots (II)$$

$$\underline{12,72 M_B} = 116,2 \rightarrow M_B = \frac{116,2}{12,72} = 9,15 \text{ tm.}$$

$$(II) \quad 1,33M_B + 4,67M_C = 0 \rightarrow 1,33 \cdot 9,15 + 4,67M_C = 0 \rightarrow M_C = -\frac{1,33 \cdot 9,15}{4,67} = -2,61 \text{ tm}$$

Muatan dianggap terletak diatas batang sendi - rol :

Perhatikan konstruksi AB : $\bar{R}_A = \frac{4 \cdot 10 + 4 \cdot 5 + 6 \cdot 3}{12} = 6,5t.$

$$\bar{R}_{B_1} = \frac{4 \cdot 2 + 4 \cdot 7 + 6 \cdot 9}{12} = 7,5t.$$

$$\bar{M}_I = \bar{R}_A \cdot 4 - 4 \cdot 2 - 6 \cdot 5 \cdot 4 - 8 = 18tm. \quad \bar{M}_{II} = \bar{R}_A \cdot 7 - 4 \cdot 5 = 6,5 \cdot 7 - 20 = 25,5tm.$$

$$\bar{M}_{III} = \bar{R}_B \cdot 3 = 7,5 \cdot 3 = 22,5tm.$$

$$b) \quad R_A = \bar{R}_A - \frac{M_B}{1} = 6,5t - \frac{9,15}{12} t = 5,738$$

$$R_B = \bar{R}_{B_1} + \frac{M_B}{1_1} - \frac{M_B}{1_2} - \frac{M_C}{1_2} = 7,5t + \frac{9,15}{12} + \frac{9,15}{8} - \frac{-2,61}{8} = 9,732t,$$

$$R_C = -\frac{M_B}{1_2} + \frac{M_C}{1_2} + \frac{M_C}{1_3} = -\frac{9,15}{8} + \frac{-2,61}{8} + \frac{-2,61}{6} = -1,905t.$$

$$R_D = -\frac{M_C}{1_3} = -\frac{-2,61}{6} = 0,435t.$$

Setelah bid D tergambar maka kita tahu bahwa M_{max} terletak 7 m di kanan A.

$$x = \frac{7 \cdot 9,15}{12} = 5,34 \text{ tm.}$$

$$\therefore M_{max} = 25,5tm - 5,34tm = 20,16 \text{ tm.}$$